

Markscheme

May 2026

**Mathematics:
applications and interpretation**

Higher level

Paper 2

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Instructions to Examiners

Abbreviations

- M** Marks awarded for attempting to use a correct **Method**.
- A** Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding **M** marks.
- R** Marks awarded for clear **Reasoning**.
- AG** Answer given in the question and so no marks are awarded.
- FT** Follow through. The practice of awarding marks, despite student errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg M1, A2*.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award **M0** followed by **A1**, as **A** mark(s) depend on the preceding **M** mark(s), if any.
- Where **M** and **A** marks are noted on the same line, *e.g. M1A1*, this usually means **M1** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and **A1** for using the **correct** values.
- Where there are two or more **A** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award **A0A1A1**.
- Where the markscheme specifies **A3**, **M2** *etc.*, do **not** split the marks, unless there is a note.
- The response to a “show that” question does not need to restate the **AG** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some students may be advantaged for that specific question item, it is likely that these students will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award **FT** marks as appropriate but do not award the final **A1** in the first part.

Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... (incorrect decimal value)	No. Last part in question.	Award A1 for the final mark (condone the incorrect further working)
2.	$\frac{35}{72}$	0.468111... (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award A0 for the final mark (and full FT is available in subsequent parts)

3 Implied marks

Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then **FT** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is **(M1)A1**, it is possible to award full marks for *their* correct answer, **without working being seen**. For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further **A** marks can be awarded for work which uses the error, but **M** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer **FT** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (e.g. probability greater than 1, $\sin \theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that students may be using an incorrect value.
- If the student’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any **FT** marks in the subsequent parts. This includes when students fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these **FT** rules will be explicitly noted on the markscheme.
- If a student makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a student incorrectly copies values or information from the question, this is a mis-read (**MR**). A student should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread and do not award the first mark, even if this is an **M** mark, but award all others as appropriate.

- If the question becomes much simpler because of the **MR**, then use discretion to award fewer marks.
- If the **MR** leads to an inappropriate value (e.g. probability greater than 1, $\sin \theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of students' own work does **not** constitute a misread, it is an error.
- If a student uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- **MR** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Students will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, etc.
- Alternative solutions for parts of questions are indicated by **EITHER . . . OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, **M** marks and intermediate **A** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the student work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however students may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come “*from the use of 3 sf values*”.

Simplification of final answers: Students are advised to give final answers using good mathematical form. In general, for an **A** mark to be awarded, arithmetic should be completed, and any

values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An

exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$.

However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^x$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and x^2+x are both acceptable.

Please note: intermediate **A** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a student has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10 Presentation of student work

Crossed out work: If a student has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the student indicates that they would like the work to be marked.

More than one solution: Where a student offers two or more different answers to the same question, an examiner should only mark the first response unless the student indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

1. (a) finding/recognizing that minimum occurs at $x = 3$ (A1)

Note: May be seen in a sketch.

1.5m

A1

Note: Award (A0)A1 if coordinates are given.

[2 marks]

- (b) $(a =) 3, (b =) 6$

A1

Note: Accept answer written as an interval.

[1 mark]

- (c) (i) $(p =) 1.5, (q =) \frac{35}{18}(1.94444\dots)$

A1A1

Note: Award A1 each correct value.

- (ii) attempt to substitute values into formula for trapezoid rule (M1)

$$\frac{1}{2} \times 1 \times \left(2.5 + 2.5 + 2 \left(\frac{35}{18} + \frac{29}{18} + 1.5 + \frac{29}{18} + \frac{35}{18} \right) \right) \quad (A1)$$

Note: As $h = 1$, accept its omission.

$$11.1 \quad \left(11\frac{1}{9}, 11.1111\dots, \frac{100}{9} \right) \text{ m}^2 \quad A1$$

[5 marks]

- (d) EITHER

the tops (legs) of the trapezoids are above the curve of the top of the gate

R1

OR

(shape is) convex OR concave up

R1

OR

function is a positive parabola

R1

Note: Accept equivalent explanations, including communicated with a diagram.
Award R0 for a response that uses their answer to part (e)(ii) to *show that* part(c)(ii) is an overestimate.

[1 mark]

(e) (i) $\int_0^6 \left(\frac{1}{9}x^2 - \frac{2}{3}x + 2.5 \right) dx$ **OR** $\int_0^6 h(x) dx$

A1A1

Note: Award **A1** for an integral of the quadratic function and **A1** for integral sign with correct limits and dx .

(ii) 11 m^2

A2

[4 marks]

(f) $\left| \frac{11\frac{1}{9} - 11}{11} \right| \times 100 \%$

(M1)

$1.01 \text{ (1.01010...) } \%$

A1

[2 marks]

Total [15 marks]

2. (a) 120 seen anywhere (A1)
 attempt to substitute into cosine rule (M1)
 $(\text{distance})^2 = 9^2 + 15^2 - 2(9)(15)\cos(120^\circ)$ (A1)
 (distance =) 21 cm A1
[4 marks]

- (b) (i) 15 A1
 (ii) recognition that period is 60 (A1)
 $\left(b = \frac{360^\circ}{60} = \right) 6$ A1
 (iii) 15 A1
[4 marks]

- (c) **EITHER**
 attempt to substitute 20 and their values into $h(t)$ (M1)
 $15\cos((6 \times 20)^\circ) + 15$
OR
 attempt to use trig ratio with angle of 30° or 60° to find a height from the centre of the clock. (M1)
 $15 - 15\sin(30^\circ)$ or $15 - 15\cos(60^\circ)$
THEN
 7.5cm A1
[2 marks]

- (d) the angle between the two hands is greater (than 120°) / has increased R1

Note: Accept equivalent responses based around the fact the hour hand moves and is not locked at 8.

[1 mark]

(e) **METHOD 1**

$$7.5 - 9 \sin(20^\circ) \quad \text{OR} \quad 7.5 - 9 \cos(20^\circ)$$

(A1) (M1)

Note: Award **(A1)** for 20° or 70° or $3.07818\dots$ seen, **(M1)** for attempt to find difference in heights.

$$4.42 \text{ cm } (4.42181\dots)$$

A1

METHOD 2

$$\text{using } h_h(t) = 9 \cos\left(\frac{1}{2}(t - 240^\circ)\right) + 15 \text{ for the height of the hour hand}$$

above the bottom of the clock.

Note: There are other possible values for the phase shift.

$$h_h(20) = 11.9218\dots$$

(A1)

attempt to find difference in heights that involves their answer from part (c)

(M1)

$$11.9218\dots - 7.5$$

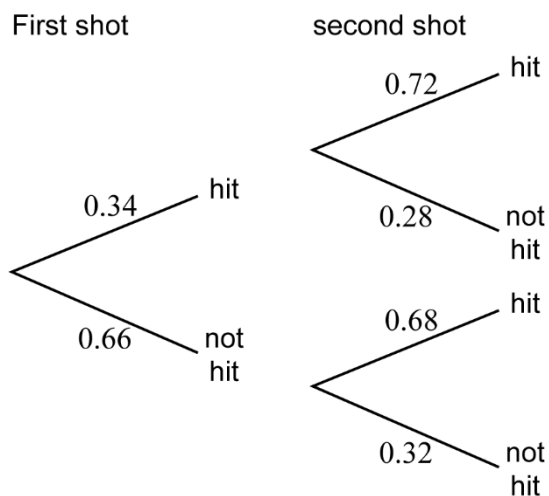
$$4.42 \text{ cm } (4.42181\dots)$$

A1

[3 marks]

Total [14 Marks]

3. (a)



A1A1A1

Note: Award **A1** for all given values, correct, on a tree diagram, **A1** for one complement value correctly placed in any branch, **A1** for the other two complement values correct. Award **A0A1A1** if labels are missing.

[3 marks]

(b) one correct product from their diagram

(M1)

$$0.34 \times 0.28 + 0.66 \times 0.68 + 0.66 \times 0.32 \quad \text{OR} \quad 1 - 0.34 \times 0.72$$

(A1)

$$= 0.755 \quad (0.7552)$$

A1

Note: Award **(M1)A0A0** for an unsupported final answer of 0.544 (exactly 1 hit). Award **M1A1A0** for 0.76.

[3 marks]

(c) $(0.34)(0.72) + (0.66)(0.68)$

(M1)

$$= 0.694 \quad \left(0.6936, \frac{867}{1250} \right)$$

A1

[2 marks]

(d) recognizing that the variable has a Binomial distribution

(M1)

$$X \sim B(10, 0.94) \quad \text{OR} \quad X \sim B(10, 0.06)$$

$$0.344 \quad (0.343796\dots)$$

A1

[2 marks]

- (e) recognizing correct cumulative probability (A1)

$$P(X \geq 7) \quad \text{OR} \quad 1 - P(X \leq 6)$$

$$0.998 \quad (0.997970\dots)$$

A1

[2 marks]

- (f) mean = 94 A1

$$\text{variance} = 100 \times 0.94 \times 0.06$$

(M1)

$$\text{SD} = 2.37 \left(2.37486\dots, \frac{\sqrt{141}}{5} \right)$$

A1

[3 marks]

- (g) $P(X \leq k) > 0.25$ (M1)

k	$P(X \leq k)$
...	
91	0.146286...
92	0.251650...
...	

Note: Award (M1) for the use of INVBINOMIAL.

$$(k =) 92$$

A1

Note: Award (M1)A0 for an answer of 91 or 93.

[2 marks]

Total [17 marks]

4. (a) (3,4)

A1A1

Note: Award **A1A0** if answer is presented as a vector but values are correct.

[2 marks]

(b) **METHOD 1**

recognition that distance is $|r|$ **OR** that Pythagoras' theorem required

(M1)

$$|r|^2 = [4\sin(2t) + 3\cos(2t)]^2 + [4\cos(2t) - 3\sin(2t)]^2$$

A1

$$= 16\sin^2 2t + 9\cos^2 2t + 16\cos^2 2t + 9\sin^2 2t$$

$$= 25(\sin^2 2t + \cos^2 2t)$$

A1

use of Pythagorean identity

M1

$$= 25$$

hence constant distance of 5 from the origin

A1

METHOD 2

$$\text{let } \mathbf{a} = \begin{pmatrix} \sin(2t) \\ \cos(2t) \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} \cos(2t) \\ -\sin(2t) \end{pmatrix}$$

$$\mathbf{r} = 4\mathbf{a} + 3\mathbf{b}$$

$$\mathbf{a} \cdot \mathbf{b} = \sin(2t)\cos(2t) - \cos(2t)\sin(2t) = 0$$

M1

Note: Award **M1** for taking scalar product of $\mathbf{a}$ and $\mathbf{b}$.

Since $\mathbf{a} \cdot \mathbf{b} = 0$, $\mathbf{a}$ and $\mathbf{b}$ are perpendicular

R1

$|4\mathbf{a}| = 4$ and $|3\mathbf{b}| = 3$ and $\mathbf{a}$ and $\mathbf{b}$ are perpendicular

M1

Note: Award **M1** for seeing both reasons.

$$|4\mathbf{a} + 3\mathbf{b}| = \sqrt{4^2 + 3^2} = 5$$

A1

hence constant distance of 5 from the origin

A1

Note: Award **(M1)A0A0M0A1** for an unsupported answer of 5 or for methods dependent on the GDC.

[5 marks]

(c) $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ (A1)

attempt at chain rule (M1)

$$= \begin{pmatrix} 8\cos(2t) - 6\sin(2t) \\ -8\sin(2t) - 6\cos(2t) \end{pmatrix} \quad \text{A1}$$

[3 marks]

(d) require $\mathbf{r} \cdot \mathbf{v} = 0$ (seen anywhere) R1

Note: This may be implied by subsequent working. If $\mathbf{r} \cdot \mathbf{v} = 0$ is not seen, a conclusion consistent with the final result must be stated.

$$\begin{aligned} & [4\sin(2t) + 3\cos(2t)] \times [8\cos(2t) - 6\sin(2t)] \\ & + [4\cos(2t) - 3\sin(2t)] \times [-8\sin(2t) - 6\cos(2t)] \end{aligned} \quad \text{(A1)}$$

attempt at expansion of brackets M1

further correct working leading to answer of zero A1

$$32cs - 24s^2 + 24c^2 - 18cs - 32cs - 24c^2 + 24s^2 + 18cs = 0$$

hence perpendicular AG

[4 marks]

Total [14 marks]

5. (a) $\frac{dy}{dt} = \frac{d^2x}{dt^2}$ A1

$\frac{dy}{dt} = -2x - 3y$ A1

$\frac{dx}{dt} = y$ AG

[2 marks]

(b) $\begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}$ A1A1

Note: Award **A1** for each correct row.

[2 marks]

(c) attempt to find eigenvalues (M1)

$$\begin{vmatrix} \lambda & -1 \\ 2 & \lambda + 3 \end{vmatrix} (= 0)$$

$\lambda^2 + 3\lambda + 2 = 0$ (A1)

$\lambda = -1, -2$ A1

[3 marks]

(d) attempt to find eigenvectors (M1)

$$\begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -1 \begin{pmatrix} x \\ y \end{pmatrix} \quad \text{OR} \quad \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -2 \begin{pmatrix} x \\ y \end{pmatrix}$$

eigenvectors: $\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ A1A1

[3 marks]

(e) **EITHER**

$x(t) = Ae^{-2t} + Be^{-t}$ A1

OR

$\begin{pmatrix} x \\ y \end{pmatrix} = Ae^{-2t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + Be^{-t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ A1

[1 mark]

(f) $\frac{dx}{dt} = -2Ae^{-2t} - Be^{-t}$

attempt to form a system of two equations

(M1)

$A + B = 20$ **or** $-2A - B = -50$

A1

Both $A + B = 20$ **and** $-2A - B = -50$

A1

$A = 30, B = -10$

$x(t) = 30e^{-2t} - 10e^{-t}$

AG

Note: Award the final **A1** only if their simultaneous equations are consistent with the given answer.

[3 marks]

(g) (i) $(t =) 1.10 \ (1.09861..., \ln(3))$ seconds

A1

(ii) $-0.833 \left(-0.833333... - \frac{5}{6} \right) \text{ m}$

A2

Note: Accept “0.833 m below the surface of the water”. Award **A1** if correct coordinate pair seen **OR** $t = 1.79176...$ seen.

[3 marks]

Total [17 marks]

6. (a) let $X(10) \sim$ number of calls Allison receives in a 10-minute period

$$X(10) \sim \text{Po}(1.25)$$

recognition that correct Poisson CDF is required using weak inequality (A1)

$$P(X(10) \geq 3) = [1 - P(X(10) \leq 2)]$$

$$= 0.132 \text{ (0.131532...)} \quad \text{A1}$$

[2 marks]

- (b) $X(20) \sim \text{Po}(2.5)$ (A1)

attempt to find $P(X(20) = 0)$ (M1)

$$= 0.0821 \text{ (0.0820849...)} \quad \text{A1}$$

[3 marks]

- (c) let $Y(30) \sim$ number of calls Allison & Jean receive in a 30-minute period

recognition that sum of means required (e.g. 2.07 seen) (M1)

$$Y(30) \sim \text{Po}(6.21) \quad \text{(A1)}$$

$$P(Y(30) < 4)$$

$$= 0.133 \text{ (0.133425...)} \quad \text{A1}$$

[3 marks]

- (d) let $C \sim$ time Allison takes to complete a call, $C \sim N(12, 9)$

attempt to find $P(C > 10)$ **OR** sketch of correct region (M1)

$$= 0.748 \text{ (0.747507...)} \quad \text{A1}$$

[2 marks]

- (e) $\bar{C} \sim N(12, 1.5)$ OR $\sum_1^6 C \sim N(72, 54)$ (A1)(A1)

Note: Award **A1** for 12 or 72 given as the mean, **A1** for 1.5 or 54 given as the variance.

$$P(\bar{C} < 10) \text{ OR } P\left(\sum_1^6 C < 60\right)$$

$$= 0.0512 \text{ (0.0512352...)} \quad \text{A2}$$

[4 marks]

(f) $P(A > J)$

EITHER

conversion to inequality with zero

(M1)

$$P(A - J > 0)$$

$$A - J \sim N(1, 13)$$

(A1)

OR

conversion to inequality with zero

(M1)

$$P(J - A < 0)$$

$$J - A \sim N(-1, 13)$$

(A1)

THEN

$$= 0.609 \quad (0.609244\dots)$$

A1

[3 marks]

Total [17 marks]

7. (a) attempt to find a common denominator M1

$$\left(\frac{A}{x} + \frac{A}{(2-x)} \right) = \frac{A(2-x) + Ax}{x(2-x)}$$

$$= \frac{2A}{x(2-x)} \quad \text{OR} \quad 1 = 2A - Ax + Ax \quad \text{seen} \quad \text{A1}$$

$$A = \frac{1}{2} \quad \text{AG}$$

[2 marks]

- (b) attempt to separate variables M1

$$\int \frac{1}{x(2-x)} dx = \int 1 dt \quad \text{A1}$$

substitution of part (a) into their rearranged equation M1

$$\frac{1}{2} \int \frac{1}{x} + \frac{1}{(2-x)} dx = \int 1 dt$$

$$\frac{1}{2} (\ln|x| - \ln|2-x|) = t + C \quad \text{OR} \quad \frac{1}{2} \left(\ln \left| \frac{x}{2-x} \right| \right) = t + C \quad \text{A1A1A1}$$

Note: Award **A1** for one $\ln$ term correct, **A1** for entire LHS correct and **A1** for $t + C$.

[6 marks]

- (c) substitution of $x = 1$, $t = 0$ into **their** part (b) M1

$$C = 0$$

$$t = \frac{1}{2} (\ln|x| - \ln|2-x|) \quad \text{OR} \quad t = \frac{1}{2} \left(\ln \left| \frac{x}{2-x} \right| \right) \quad \text{A1}$$

Note: If C is not seen in part (b) award **M0A0**. Do not condone missing modulus signs.

[2 marks]

- (d) **EITHER**
 x and $2-x$ are **both** positive (in this interval) R1

OR

$$\frac{x}{2-x} \text{ is positive (in this interval)} \quad \text{R1}$$

[1 mark]

(e) $t = \frac{1}{2}(\ln x - \ln(2-x))$

express RHS as a single logarithm (may be seen in earlier parts)

(M1)

$$t = \frac{1}{2} \left(\ln \frac{x}{2-x} \right)$$

attempt to rewrite their equation to include an exponential function

(M1)

$$\frac{x}{2-x} = e^{2t}$$

(A1)

remove fractional term and expand brackets

(A1)

$$x = 2e^{2t} - xe^{2t}$$

$$x(1 + e^{2t}) = 2e^{2t}$$

$$x = \frac{2e^{2t}}{1 + e^{2t}}$$

A1

[5 marks]

Total [16 Marks]
